4 - Sets 1

Definition. A **set** is an unordered collection of objects (with no repeats), written in braces, like:

Students = {Ayo, Ian, Elon} Faculty = {Man, Elon} Staff = {Neil, Elon}

An object x is an **element** or **member** of a set S, written x∈S, if x is listed within the outer curly braces of S:

Elon ∈ Students, Man ∉ Students

A set S is **subset** of a set T, written S⊆T, if x∈S satisfies x∈T.

Application. Sets model group permissions:

Students ∪ Staff = {Ayo, Ian, Elon} get gym access

Faculty ∪ Staff = {Man, Neil, Elon} get weekend building access

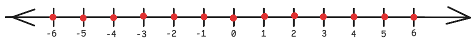
Students ∩ Faculty ∩ Staff = {Elon} lists suspicious users (too much access)

Definition.

The empty set, written as ∅ or {} has no elements. This set is unique.

$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$



$\mathbb{Q} =$ the set of rational numbers, whose elements are quotients $\dfrac{n}{m}$ of integers n and m with $m \neq 0$

$\mathbb{R} =$ the set of real numbers

Know: $\dfrac{2}{5} = 0.4 \in \mathbb{Q}, \quad \sqrt{2} \notin \mathbb{Q}, \quad \sqrt{2} \in \mathbb{R}, \quad \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

Definition. (Set-roster notation.)
Let U be a set of all possible elements under consideration, called the **universe**. Then

$$\{x \in U : P(x)\}$$

is the set of all elements x of U such that the statement P(x) about x is true.
We read the colon as "such that".

Examples.

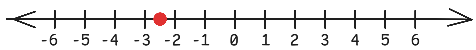
(1) $\mathbb{Q} = \left\{ \dfrac{n}{m} : n, m \in \mathbb{Z} \text{ and } m \neq 0 \right\}$

(2) The set of 2-digit square numbers = $\{n \in \mathbb{N} : \sqrt{n} \in \mathbb{N} \text{ and } 10 \leq n \leq 99\}$

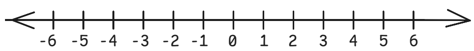
= $\{16, 25, 49, 64, 81\}$

Examples. List all elements of the following sets; graph the sets on the number line.

(1) $\{x \in \mathbb{Q} : 2x + 5 = 0\} = \{-5/2\}$ by solving 2x+5=0.



(2) $\{x \in \mathbb{Z} : 2x + 5 = 0\} = \{\}$: -5/2=-2.5 is not an integer.



③ $\{x \in \mathbb{Q} : x^2 - 2 = 0\} = \{\}$ since $\pm\sqrt{2}$ are not rational numbers.

④ $\{x \in \mathbb{Q} : 2x^3 - x^2 - 4x + 2 = 0\}$

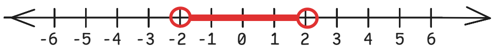
> **Rational Roots Test:** For a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with $a_i \in \mathbb{Z}$ a zero $\frac{p}{q} \in \mathbb{Q}$ of $f(x)$ satisfies $p \mid a_0$ and $q \mid a_n$.

In our case, p|2 so p∈{±1,±2} and q|2 so q∈{±1,±2}. So $\frac{p}{q} \in \left\{\pm 1, \pm\frac{1}{2}, \pm 2\right\}$

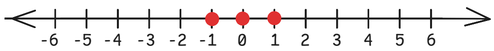
Checking by hand, only x = 1/2 is a solution to our polynomial.

Examples. Graph the following sets on the number line.

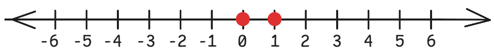
① $\{x \in \mathbb{R} : x^2 < 4\}$



② $\{x \in \mathbb{Z} : x^2 < 4\}$



③ $\{x \in \mathbb{N} : x^2 < 4\}$



Definition.

$(a, b) = \{x \in \mathbb{R} : a < x \text{ and } x < b\}$, called the **open interval** from a to b

$[a, b] = \{x \in \mathbb{R} : a \leq x \text{ and } x \leq b\}$, called the **closed interval** from a to b

Definition. Let A and B be subsets of the universe U.

$\sim A = \{x \in U : x \notin A\}$ is the **complement** of A in U.

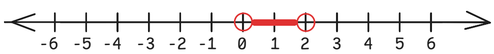
$A \cup B = \{x \in U : x \in A \text{ or } x \in B\}$ is the **union** of A and B.

$A \cap B = \{x \in U : x \in A \text{ and } x \in B\}$ is the **intersection** of A and B.

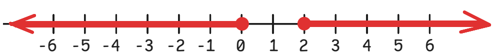
$A - B = \{x \in U : x \in A \text{ and } x \notin B\}$ is the **difference** of A and B.

Examples. Graph the following sets on the number line.

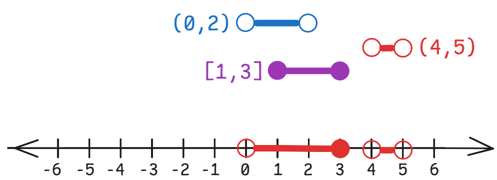
-1 $(0, 2)$



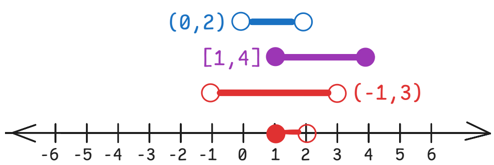
0 $\sim (0, 2)$



① $(0, 2) \cup (4, 5) \cup [1, 3]$



② $(0, 2) \cap [1, 4] \cap (-1, 3)$



③ $(\sim (0, 2)) \cap [1, 3]$